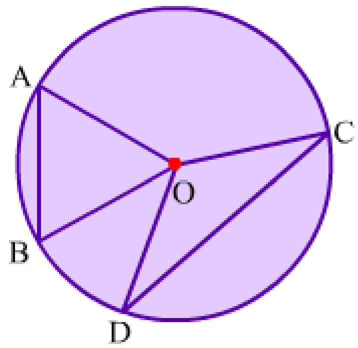
# Circles

## Angles Subtended by Chords at The Centre

We know that the chord of a circle is a line segment having its endpoints on the circumference of the circle. There can be several chords of the same or different lengths in a circle. Observe, for example, two unequal chords of a circle with centre O.




In the circle, chord CD is longer than chord AB. Note the angles subtended by the chords at the centre. While AB subtends an acute angle (i.e., ∠AOB), CD subtends an obtuse angle (i.e., ∠COD). Thus, in a circle, chords of different lengths subtend different angles at the centre.

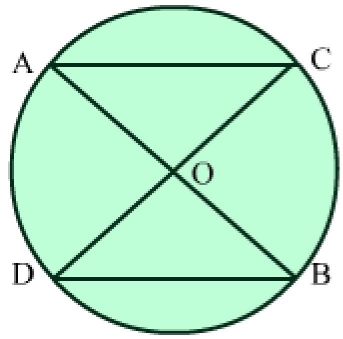
On the other hand, **equal (or congruent) chords subtend equal (or congruent) angles at the centre**. This is a very useful property of circle. In this lesson, we will learn more about this property and solve some examples based on the same.

| **Know More** |
|---|
| The diameter of a circle is the longest chord of the circle. |

**Solved Examples**

**Easy**

**Example 1:** AB and CD are diameters of the given circle. Show that AC and BD are equal chords of the circle.




**Solution:**

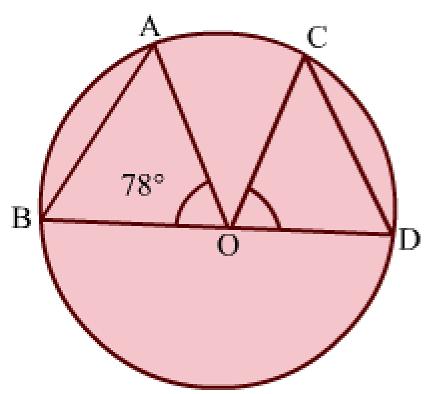
Since AB and CD are diameters of the circle, their point of intersection is the centre of the circle. Thus, O is the centre of the circle.

Chord AC subtends ∠AOC at the centre, while chord BD subtends ∠BOD at the centre.

Now, ∠AOC = ∠BOD (Vertically opposite angles)

∴ AC = BD (∵ Chords subtending equal angles at the centre are equal in length)

**Example 2**: AB and CD are equal chords of the given circle with centre O. Find the measure of ∠COA.




**Solution**:

It is given that AB and CD are equal chords of the circle.

∴ ∠AOB = ∠COD = 78° (∵ Equal chords subtend equal angles at the centre)

Now,

∠AOB + ∠COA + ∠COD = 180° (∵ BOD is the diameter of the circle)

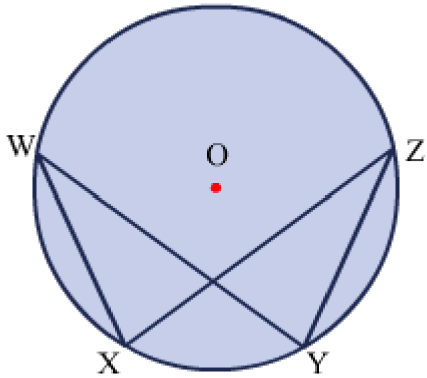
⇒ 78° + ∠COA + 78° = 180°

⇒ ∠COA = 180° − (78° + 78°)

⇒ ∠COA = 24°

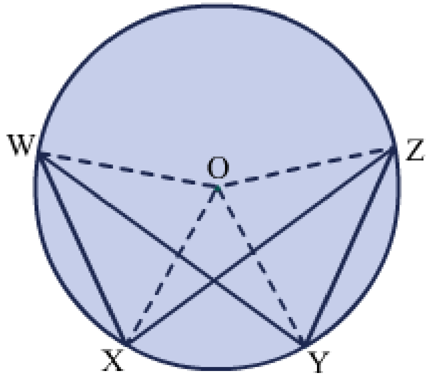
**Medium**

**Example 1**: If WY and ZX are equal chords of the given circle with centre O, then show that WX and ZY are also equal chords.




**Solution**:

***Construction***: Join O to W, X, Y and Z.




It is given that WY and ZX are equal chords.

∴ ∠WOY = ∠ZOX (∵ Equal chords subtend equal angles at the centre)

On subtracting ∠XOY from both sides of the equation, we obtain:

∠WOY −∠XOY = ∠ZOX − ∠XOY

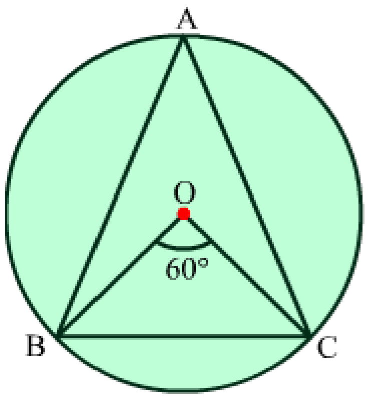
⇒ ∠WOX = ∠ZOY

Now, ∠WOX and ∠ZOY are the angles subtended by chords WX and ZY at the centre.

∴ WX = ZY   (∵ Chords subtending equal angles at the centre are equal in length)

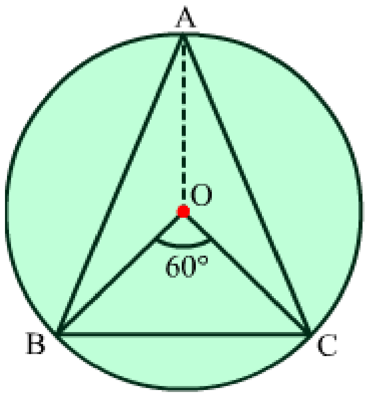
**Hard**

**Example 1**: In the given circle with centre O, AB = AC. Find the measure of ∠BAC.




**Solution**:

***Construction***: Join O to A.




It is given that AB = AC.

∠AOB = ∠AOC (∵ Equal chords subtend equal angles at the centre)

Let ∠AOB = ∠AOC = $x$

Now,

$\angle AOB + \angle BOC + \angle AOC = 360°$ (Complete angle)

$\Rightarrow x + 60° + x = 360°$

$\Rightarrow 2x + 60° = 360°$

$\Rightarrow 2x = 360° - 60°$

$\Rightarrow 2x = 300°$

$\Rightarrow x = 150°$… (1)

Now, in ΔOAB, we have:

OA = OB (Radii of the circle)

$\Rightarrow \angle OBA = \angle OAB$ (Angles opposite equal sides are equal)

Using the angle sum property in ΔOAB, we get:

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 2\angle OAB + 150° = 180°$ (By equation 1)

$\Rightarrow 2\angle OAB = 180° - 150°$

$\Rightarrow 2\angle OAB = 30°$

$\Rightarrow \angle OAB = 15°$

Similarly, in ΔOAC, we get $\angle OAC = 15°$

Now, $\angle BAC = \angle OAB + \angle OAC$

$\Rightarrow \angle BAC = 15° + 15°$

$\Rightarrow \angle BAC = 30°$

**Perpendicular from The Centre of a Circle to a Chord Bisects The Chord**

**Perpendicular from the Centre to the Chord**

Observe the clock shown below.


Note how the progression of the second hand from '5' to '7' has been marked on the clock face. We know that the second hand takes five seconds to cover the distance between '5' and '6'. Similarly, it takes five seconds to cover the distance between '6' and '7'. Clearly, '6' lies in the middle of '5' and '7'. Observe how the line marking the position of the second hand at '6' is perpendicular to the straight line marking the distance between '5' and '7'. What we have here is a system similar to that obtained on drawing a perpendicular from the centre of a circle to one of its chords.

In this lesson, we will learn about the property of a circle relating to perpendiculars drawn to chords from the centre of the circle. We will also solve some problems based on this property.

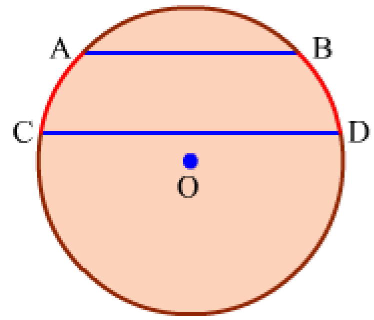
**Know More**

The circle that is centred at the origin with radius 1 is called the **unit circle**.

**Whiz Kid**

- For a given length of perimeter, the circle is the shape with the largest area.
- The circle is a highly symmetric shape as every line passing through its centre forms a line of reflection symmetry and every angle around the centre has rotational symmetry.
- In a circle, parallel chords always cut congruent arcs.

Here, chord AB is parallel to chord CD and arc AC and arc BD are congruent arcs.



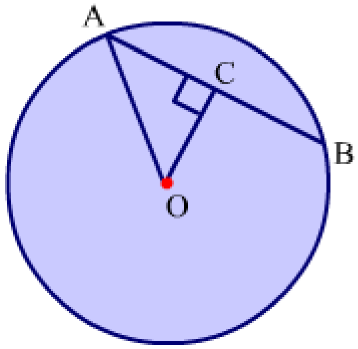

**Whiz Kid**

If the intersection of any two chords divides one chord into lengths *a* and *b* and the other into lengths *c* and *d*, then *ab* = *cd*.

**Solved Examples**

**Easy**

**Example 1**: In the given circle with centre O, if AB = 6 cm and OC = 4 cm, then find the perimeter of ΔOCA.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OC ⊥ AB; therefore, OC bisects AB.

∴ **AC = CB =** $\frac{AB}{2} = \left(\frac{6}{2}\right)\text{cm} = 3$ cm

On using the Pythagoras theorem in the right-angled ΔOCA, we get:

$OA^2 = OC^2 + AC^2$

$\Rightarrow OA^2 = (4^2 + 3^2)\ cm^2$

$\Rightarrow OA^2 = (16 + 9)\ cm^2$

$\Rightarrow OA^2 = 25\ cm^2$

$\Rightarrow OA = 5$ cm

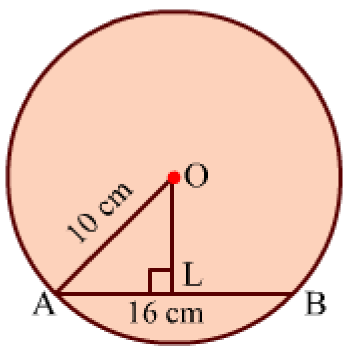
So,

Perimeter of ΔOCA = OA + AC + OC

⇒ Perimeter of ΔOCA = (5 + 3 + 4) cm

⇒ Perimeter of ΔOCA = 12 cm

**Example 2**: In the given circle centred at O, find the distance of chord AB from the centre.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OL ⊥ AB; therefore, OL bisects AB.

$\Rightarrow AL = LB = \frac{AB}{2} = \left(\frac{16}{2}\right) cm = 8$ cm

On using the Pythagoras theorem in the right-angled ΔOLA, we get:

$OA^2 = OL^2 + AL^2$

$\Rightarrow OL^2 = OA^2 - AL^2$

$\Rightarrow OL^2 = (10^2 - 8^2)\ cm^2$

$\Rightarrow OL^2 = (100 - 64)\ cm^2$

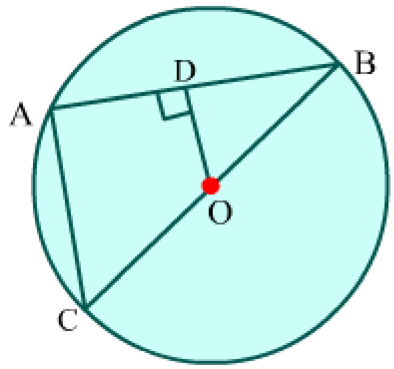
$\Rightarrow OL^2 = 36\ cm^2$

$\Rightarrow OL = 6\ cm$

Thus, chord AB is at a distance of 6 cm from the centre of the circle.

**Medium**

**Example 1:** In the given circle with centre O, prove that AC = 2OD.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OD ⊥ AB; therefore, OD bisects AB.

Thus, D is the midpoint of AB.

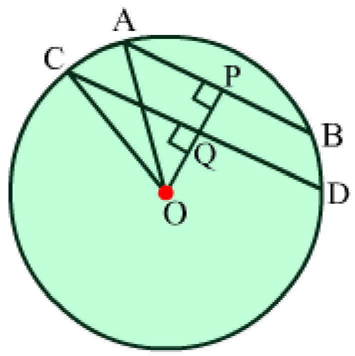
Since O is the centre, it is the midpoint of diameter BC.

So, in ΔABC, D and O are the midpoints of sides AB and BC respectively.

$\therefore OD = \frac{AC}{2}$ and OD||AC (By the midpoint theorem)

$\Rightarrow AC = 2OD$

**Example 2**: The given circle centred at O has a radius of 5 cm and two parallel chords AB and CD. If AB = 6 cm and CD = 8 cm, then find the length of PQ.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OP ⊥ AB; therefore, OP bisects AB.

Thus, P is the midpoint of AB.

$\Rightarrow AP = PB = \frac{AB}{2} = \left(\frac{6}{2}\right)\text{cm} = 3 \text{ cm}$

Similarly, Q is the midpoint of CD.

$\Rightarrow CQ = QD = \frac{CD}{2} = \left(\frac{8}{2}\right)\text{cm} = 4 \text{ cm}$

On using the Pythagoras theorem in right-angled ΔOPA, we get:

$OA^2 = OP^2 + AP^2$

$\Rightarrow OP^2 = OA^2 - AP^2$

$\Rightarrow OP^2 = (5^2 - 3^2) \text{ cm}^2$

$\Rightarrow OP^2 = (25 - 9) \text{ cm}^2$

$\Rightarrow OP^2 = 16 \text{ cm}^2$

$\Rightarrow OP = 4 \text{ cm}$

Similarly, in right-angled ΔOQC, we get:

$OC^2 = OQ^2 + CQ^2$

$\Rightarrow OQ^2 = OC^2 - CQ^2$

$\Rightarrow OQ^2 = (5^2 - 4^2)\ cm^2$

$\Rightarrow OQ^2 = (25 - 16)\ cm^2$

$\Rightarrow OQ^2 = 9\ cm^2$

$\Rightarrow OQ = 3$ cm

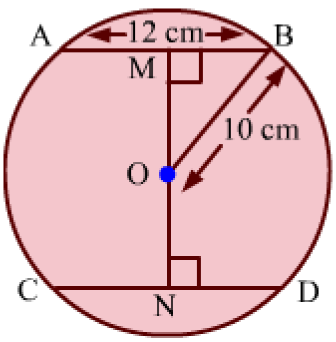
Now, PQ = OP − OQ

$\Rightarrow PQ = (4 - 3)$ cm

$\Rightarrow PQ = 1$ cm

**Hard**

**Example 1:** In the given circle with centre O, find the distance between the parallel and equal chords AB and CD.




**Solution:**

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OM ⊥ AB; therefore, OM bisects AB.

$\Rightarrow AM = MB = \frac{AB}{2} = \left(\frac{12}{2}\right)cm = 6$ cm

Similarly, ON bisects CD. Since AB = CD, we get:

CN = ND = 6 cm

On using the Pythagoras theorem in right-angled ΔOMB, we get:

$OB^2 = OM^2 + BM^2$

$\Rightarrow OM^2 = OB^2 - BM^2$

$\Rightarrow OM^2 = (10^2 - 6^2)\ cm^2$

$\Rightarrow OM^2 = (100 - 36)\ cm^2$

$\Rightarrow OM^2 = 64\ cm^2$

$\Rightarrow$ OM = 8 cm

On joining point O to point D and calculating as above, we get ON = 8 cm.

Now, MN is the perpendicular distance between the given chords.

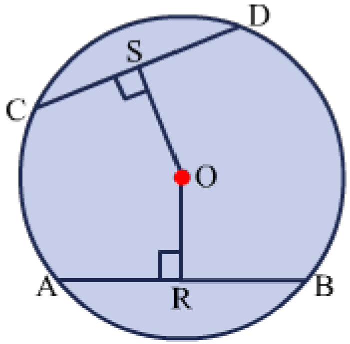
MN = OM + ON

$\Rightarrow$ MN = (8 + 8) cm

$\Rightarrow$ MN = 16 cm

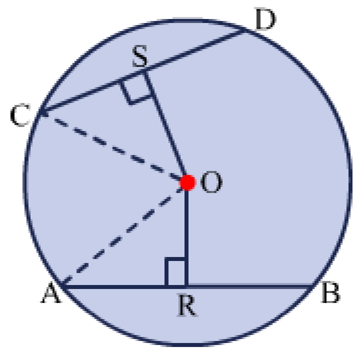
Thus, the distance between the parallel and equal chords AB and CD is 16 cm.

**Example 2**: In the given circle centred at O, AB = 8 cm, OR = 3 cm and OS = 4 cm. Find the length of CD.




**Solution:**

***Construction***: Join point O to points A and C.




We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the circle with chord AB and OR ⊥ AB; therefore, OR bisects AB.

$\Rightarrow AR = RB = \frac{AB}{2} = \left(\frac{8}{2}\right)cm = 4\ cm$

On using the Pythagoras theorem in right-angled ΔORA, we get:

$OA^2 = OR^2 + AR^2$

$\Rightarrow OA^2 = (3^2 + 4^2)\ cm^2$

$\Rightarrow OA^2 = (9 + 16)\ cm^2$

$\Rightarrow OA^2 = 25\ cm^2$

$\Rightarrow OA = 5\ cm$

Clearly, OA and OC are radii of the same circle.

$\therefore OC = OA = 5\ cm$

On using the Pythagoras theorem in right-angled ΔOSC, we get:

$OC^2 = OS^2 + CS^2$

$\Rightarrow CS^2 = OC^2 - OS^2$

$\Rightarrow CS^2 = (5^2 - 4^2)\ cm^2$

$\Rightarrow CS^2 = (25 - 16)\ cm^2$

⇒ $CS^2 = 9 \text{ cm}^2$

⇒ CS = 3 cm

Since OS ⊥ CD, we have:

CS = SD = 3 cm

Now, CD = CS + SD

⇒ CD = (3 + 3) cm

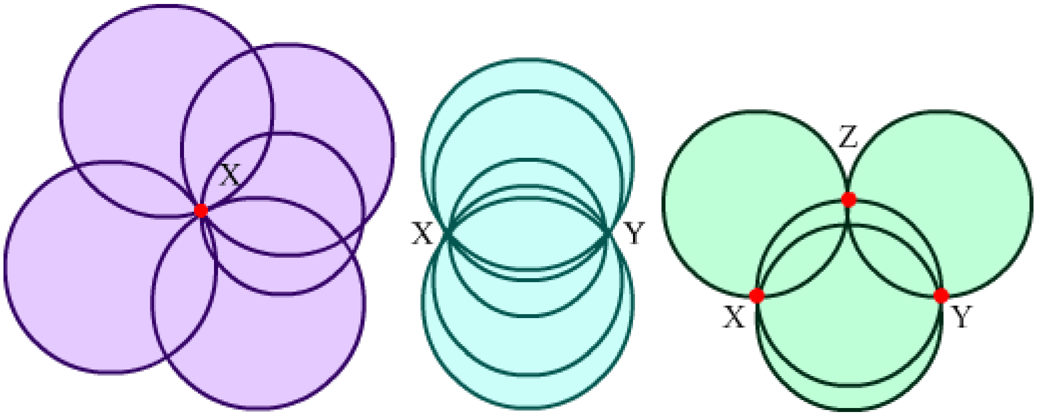
⇒ CD = 6 cm

**Only One Circle can Pass Through Three Non-Collinear Points**

**Limitation of Points Shared by Circles**

We know that the most important point required to draw a circle is its centre which is equidistant from all other points lying on the boundary of the circle. We can also draw infinitely many circles of different radii with the same centre.

Now, let us observe some points shared by circles on their boundaries. A few circles passing through common points X, Y and Z are shown below.



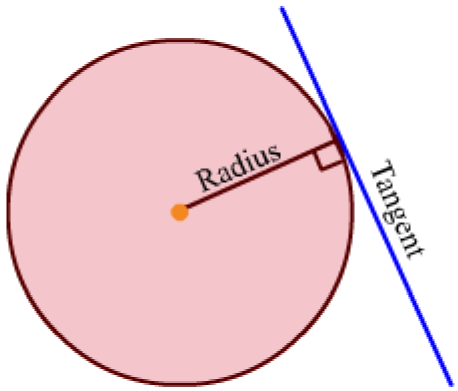

It can be observed that when point X is taken alone, we can draw infinitely many circles passing through it. Similarly, when X and Y are taken together, we can get infinitely many circles passing through them. However, when we take the three points X, Y and Z together, we obtain only one circle passing through them. Thus, we can conclude that to draw a unique circle, we require at least three non-collinear points. In this lesson, we will study more about this conclusion.

**Did You Know?**

- Circles were worshiped in ancient Rome as they were thought to be divine and holy.
- A circle is a shape that does not exist in nature. It is a mental construct and a symbolic representation that was invented in a manner similar to how the alphabet and language were invented.

**Whiz Kid**

A tangent is a line that touches a circle at only one point. It always forms a right angle with a radius of that circle.



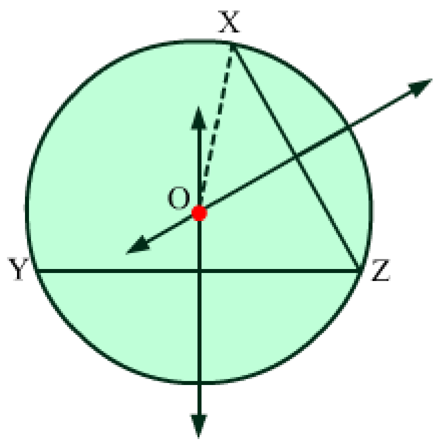

**Know More**

Two lines cannot intersect at more than one point.

**Verification of the Uniqueness of a Circle**

Let us verify the fact that only one circle passes through three non-collinear points.

The given circle with centre O passes through points X, Y and Z.



Let us assume that another circle with centre O′ and the same radius can pass through X, Y and Z. Then, O′ must lie on the perpendicular bisectors of XZ and YZ.

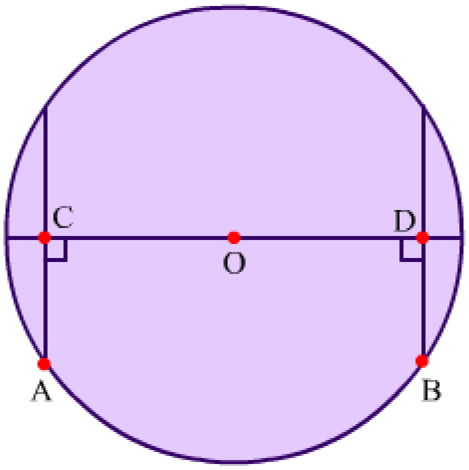
We know that two lines cannot intersect at more than one point. So, O′ must coincide with O.

Hence, one and only one circle can pass through three non-collinear points.

**Distance of Chords from The Centre of The Circle**

**Equal Chords and Their Distance from the Centre**

Consider a big circular ground in which three ropes are tied to the circumference. Two ropes are of the same length and the third is perpendicular to them, as is shown in the following figure.

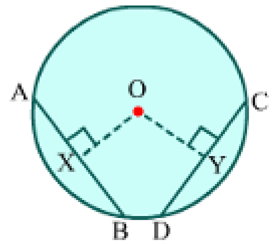



Anita, Bindu and Omkar are standing at points A, B and O respectively. The girls begin walking along the lengths of the shorter ropes to reach points C and D at the same time. Thereafter, they continue in a similar manner toward point O with the same speed as before. Who will reach Omkar first?

The answer to the above question is based on an important property of chords which we will study in this lesson. This property shows the relation between equal chords in terms of their distances from the centre of a circle.

**Equal Chords Are Equidistant from the Centre of a Circle**

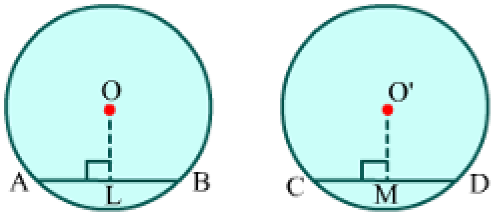
Consider the given circle centred at O with two equal chords AB and CD.




OX and OY are the perpendicular distances of chords AB and CD respectively from centre O. Now, the property that relates these perpendicular distances of equal chords is stated as follows:

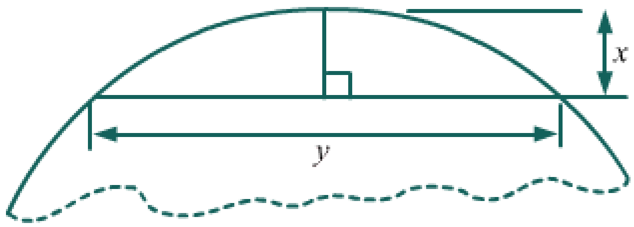
***Equal chords are equidistant from the centre of a circle.***

So, according to this property, since AB is equal to CD, their distances from the centre are also equal, i.e., OX = OY. This property is also true in case of congruent circles. Consider, for example, the following congruent circles with centres O and O′, and chords AB and CD. Now, if AB and CD are equal, then AB and CD are equidistant from O and O′, i.e., OL = O′M.




**Know More**

**Sagitta**: It is the perpendicular drawn from an arc of a circle to a chord such that it bisects the chord.

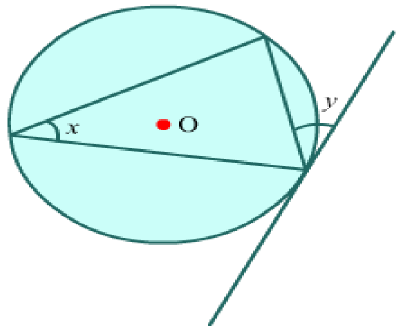



Here, $x = r - \sqrt{r^2 - \left(\frac{y}{2}\right)^2}$, where *r* is the radius of the circle

**Whiz Kid**

**Alternate segment theorem**

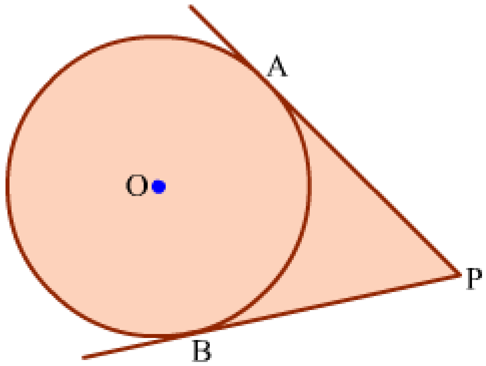
If a line touches a circle and a chord is drawn from the point of contact, then the angle between the tangent and the chord is equal to the angles in the corresponding alternate segments.




Here, $x = y$

**Whiz Kid**

The lengths of tangents drawn from a point outside a circle are equal.



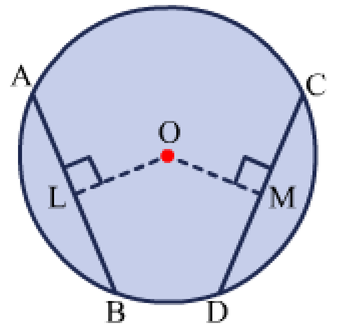

Here, PA = PB

**Converse of the Property**

The converse of the property can be stated as follows:

***Chords that are equidistant from the centre of a circle are equal in length.***

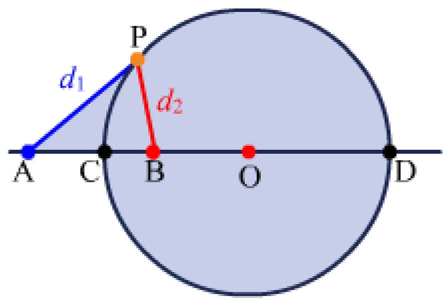
Consider, for example, the given circle with centre O and chords AB and CD that are equidistant from O, i.e., OL = OM.




Using the converse of the property, we can say that AB and CD are equal in length.

**Did You Know?**

**Circle of Apollonius**



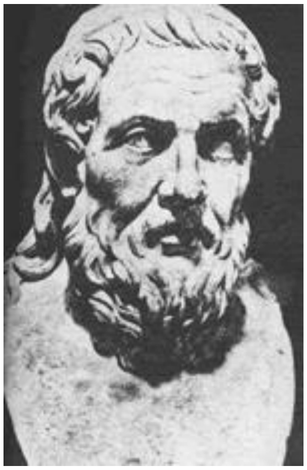

Apollonius of Perga, an ancient Greek geometer, showed that a circle can be defined as a set of points in a plane having a constant ratio of distances to two fixed foci.

In the given figure, points A and Bare two fixed foci.

Therefore, by the above definition, we have $\frac{AP}{BP} = \frac{AC}{BC}$

Note that the constant ratio cannot be equal to 1.

**Know Your Scientist**



Apollonius (262 BC−190 BC) was an ancient Greek geometer and astronomer. He is known as 'the Great Geometer'. His book *Conics* is one of the greatest scientific works from the ancient world. Terms such as 'parabola', 'ellipse' and 'hyperbola' were introduced in this book. Famous scholars like René Descartes and Isaac Newton were influenced by his innovative methods and the terms he used in the field of conics.

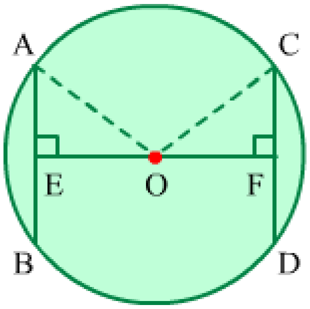
**Proof of the Converse**

**Statement:** Chords that are equidistant from the centre of a circle are equal in length.

**Given**: A circle centred at O with chords AB and CD and OE = OF, where OE ⊥AB and OF ⊥ CD.

**To Prove:** AB = CD

***Construction***: Join point O to points A and C.




**Proof:** It is given that OE ⊥ AB and OF ⊥ CD

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord. So, we have

$AE = \frac{AB}{2}$ **... (1)**

$CF = \frac{CD}{2}$ **... (2)**

In ΔAEO and ΔCFO, we have:

OA = OC (Radii of the circle)

∠AEO = ∠CFO = 90° (∵ OE and OF are perpendiculars)

OE = OF (Given)

∴ ΔAEO ≅ ΔCFO (By the RHS congruence rule)

⇒ AE = CF (By CPCT)

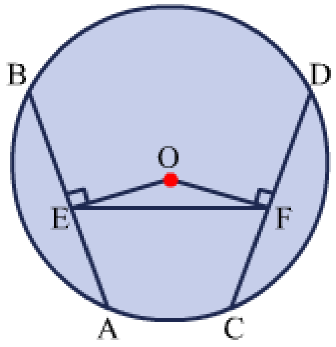
⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$ CD (Using equations (1) and (2))

⇒ AB = CD

**Solved Examples**

**Easy**

**Example 1**: In the given circle centred at O, AB and CD are two equal chords. Prove that ∠AEF = ∠CFE.




**Solution**:

It is given that AB = CD. We know that equal chords are equidistant from the centre of a circle.

∴ OE = OF

In ΔOEF, we have:

OE = OF

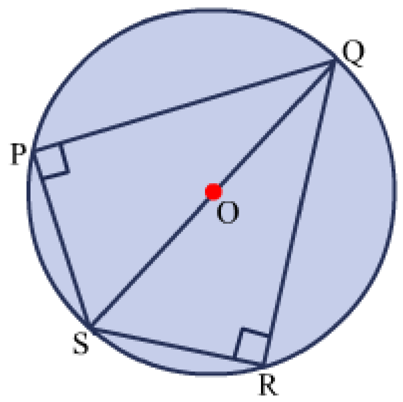
∴ ∠OFE = ∠OEF (∵ Angles opposite equal sides are equal)

⇒ 90° – ∠AEF = 90° – ∠CFE (∵ OE and OF are perpendiculars)

⇒ ∠AEF = ∠CFE

**Medium**

**Example 1**: In the given circle with centre O, chords PQ and RQ are equidistant from the centre. Prove that diameter SQ bisects ∠PQR and ∠PSR.




**Solution**:

We know that chords which are equidistant from the centre of a circle are equal in length.

∴ PQ = RQ

In ΔQPS and ΔQRS, we have:

PQ = RQ (Proved above)

∠QPS = ∠QRS = 90° (∵ SP and SR are perpendiculars)

QS = QS (Common side)

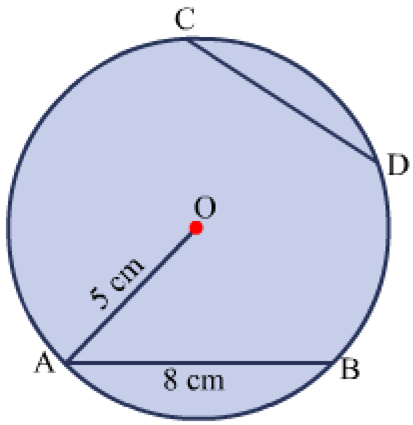
∴ ΔQPS ≅ ΔQRS (By the RHS congruence rule)

⇒ ∠PQS = ∠RQS and ∠PSQ = ∠RSQ (By CPCT)

Now, ∠PQR = ∠PQS + ∠RQS and ∠PSR = ∠PSQ + ∠RSQ

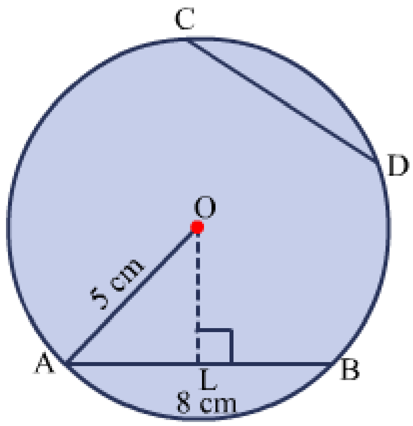
Thus, diameter QS bisects ∠PQR and ∠PSR.

**Example 2**: In the given circle, find the length of chord CD which is 3 cm away from centre O.




**Solution**:

***Construction***: Draw a perpendicular OL from centre O to chord AB.




We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ AL = LB = $\frac{AB}{2} = \left(\frac{8}{2}\right)\text{cm} = 4$ cm

On using the Pythagoras theorem in right-angled ΔOLA, we obtain:

$OA^2 = OL^2 + AL^2$

$\Rightarrow OL^2 = OA^2 - AL^2$

$\Rightarrow OL^2 = (5^2 - 4^2)\ \text{cm}^2$

$\Rightarrow OL^2 = (25 - 16)\ \text{cm}^2$

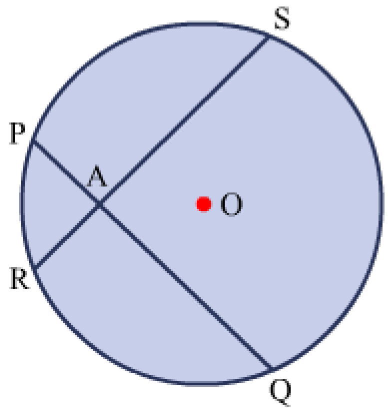
⇒ $OL^2 = 9 \text{ cm}^2$

⇒ OL = 3 cm, which is the distance of chord AB from O

It is given that the distance of chord CD from the centre is also 3 cm. We know that chords which are equidistant from the centre are equal in length. Therefore, the length of chord CD is also 8 cm.

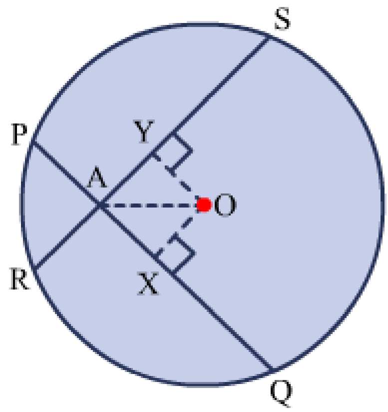
**Hard**

**Example 1**: In the given circle, PQ and RS are two chords equidistant from centre O and A is the point of intersection of the chords. Prove that AR = AP.




**Solution**:

***Construction***: Draw perpendiculars OX and OY to chords PQ and RS respectively. Join O to A.




In ΔOXA and ΔOYA, we have:

OX = OY (∵ PQ and RS are equidistant from the centre)

∠OXA = ∠OYA = 90° (∵ OX and OY are perpendiculars)

OA = OA (Common side)

∴ ΔOXA ≅ ΔOYA (By the RHS congruence rule)

⇒ AX = AY … (1) [By CPCT]

We know that chords which are equidistant from the centre are equal in length.

∴ PQ = RS … (2)

We also know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore PX = \frac{PQ}{2}$ and $RY = \frac{RS}{2}$

Using equation 2, we obtain:

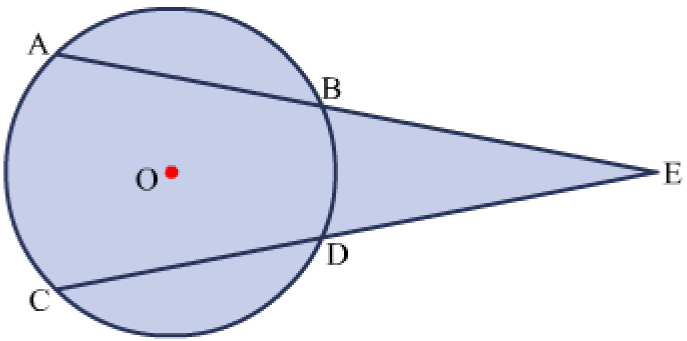
PX = RY … (3)

On subtracting equation 1 from equation 3, we obtain:

PX − AX = RY − AY

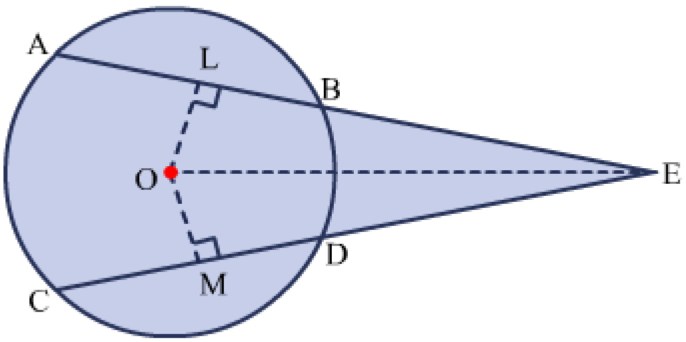
⇒ AP = AR

**Example 2**: When two equal chords AB and CD of a circle with centre O are extended, they meet at a point E as is shown in the figure. Prove that BE = DE and AE = CE.




**Solution**:

***Construction***: Join O to E and construct OL ⊥ AB and OM ⊥ CD.




It is given that AB = CD. We know that equal chords are equidistant from the centre.

∴ OL = OM

In ΔOLE and ΔOME, we have:

OL = OM (Proved above)

∠OLE = ∠OME = 90° (∵ OL and OM are perpendiculars)

OE = OE(Common side)

∴ ΔOLE ≅ ΔOME  (By the RHS congruence rule)

⇒ LE = ME … (1)  [By CPCT]

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord. Thus L and M are the midpoints of AB and CD respectively.

∴ BL = DM… (2)  [∵ AB = CD]

On subtracting equation 2 from equation 1, we get:

LE − BL = ME − DM

⇒ BE = DE

Now, AB = CD and BE = DE

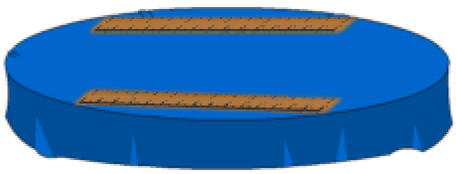
∴ AB + BE = CD + DE

⇒ AE = CE

**Relation Between The Lengths of Chords and Their Corresponding Arcs**

**Observing Chords and Their Corresponding Arcs**

Mathematics is an inseparable part of our life. It is hidden in various things around us; we just need to observe the same. For instance, look at the top of a table shown below. Two rulers of equal length are placed near the edge.



Clearly, the top of the table is circular and the two rulers near the edge resemble two chords drawn on the opposite sides of a circle. Can you notice the two arcs formed by the rulers? What do you observe about the lengths of these arcs? Finally, what can be concluded from this about the relation between the lengths of chords of a circle and their corresponding arcs?

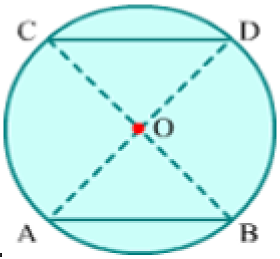
Let us go through this lesson to learn about the property of chords that relates their lengths to their corresponding arcs. We will also test the validity of this property and solve some problems based on it.

**Proof of the Property**

**Statement**:If two chords of a circle are equal, then their corresponding arcs are congruent.

**Given**: A circle with centre O and two equal chords AB and CD

**To prove**: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$




***Construction*****:** Join point O to points A, B, C and D.

**Proof**: In ΔOAB and ΔOCD, we have:

AB = CD (Given)

OA = OD (Radii of the circle)

OB = OC (Radii of the circle)

∴ ΔOAB ≅ ΔOCD (By the SSS congruence rule)

⇒ ∠AOB = ∠COD (By CPCT)

⇒ Length of $\overset{\frown}{AB}$ = Length of $\overset{\frown}{CD}$

⇒ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

**Know More**

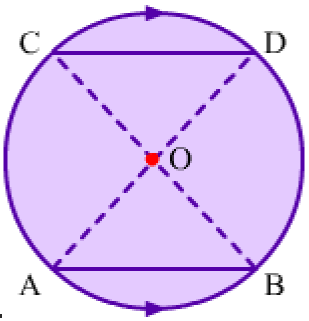
Length of an arc = $\dfrac{\text{Central angle}}{360^\circ} \times \text{Circumference}$

**Proof of the Converse**

**Statement**: If two arcs of a circle are congruent, then their corresponding chords are equal.

**Given**: A circle with centre O and two congruent arcs AB and CD

**To prove**: Chord AB = Chord CD




***Construction*****:** Join point O to points A, B, C and D.

**Proof**: It is given that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

⇒ Length of $\overset{\frown}{AB}$ = Length of $\overset{\frown}{CD}$

⇒ ∠AOB = ∠COD

In ΔOAB and ΔOCD, we have

OA = OD (Radii of the circle)

∠AOB = ∠COD (Proved above)

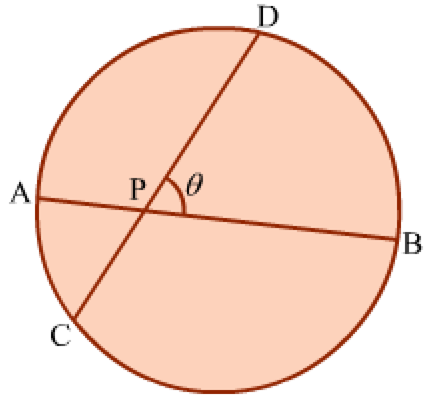
OB = OC (Radii of the circle)

∴ ΔOAB ≅ ΔOCD (By the SAS congruence rule)

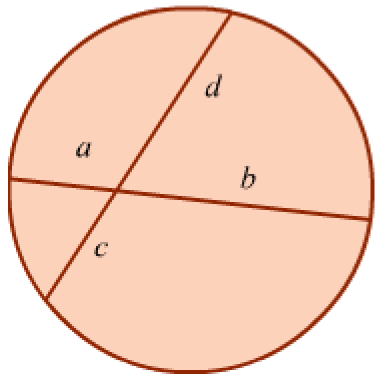
⇒ AB = CD (By CPCT)

**Whiz Kid**

- Two intersecting chords determine four arcs in a circle. The relation between the angle of intersection of the two chords and the lengths of these arcs is given as $\theta = \frac{m\overset{\frown}{AC} + m\overset{\frown}{BD}}{2}$.



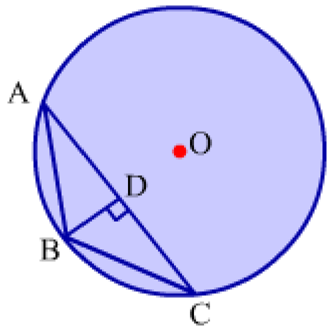

- If two chords of a circle intersect such that one chord has sub-segment lengths *a* and *b* and the other has sub-segment lengths *c* and *d*, then *ab* = *cd*.




**Solved Examples**

**Easy**

**Example 1**: In the given circle, find the length of AD if arcs AB and BC are congruent and AC = 5 cm.




**Solution:**

We know that if two arcs are congruent, then their corresponding chords are equal.

Since arcs AB and BC are congruent, chords AB and BC are equal.

⇒ ΔABC is isosceles.

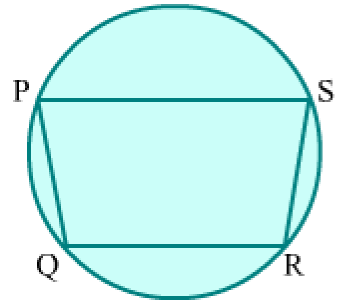
We also know that the perpendicular drawn from the vertex to the base of an isosceles triangle bisects the base.

∴ AD = DC = $\frac{1}{2}AC$

⇒ AD = $\frac{5}{2}$ cm

⇒ AD = 2.5 cm

**Example 2**: In the given circle, arc QPS is congruent to arc RSP. Prove that PQ = SR.




**Solution**: It is given that $\overset{\frown}{QPS} \cong \overset{\frown}{RSP}$

$\Rightarrow \overset{\frown}{QPS} = \overset{\frown}{RSP}$

On subtracting $\overset{\frown}{PS}$ from both sides, we get:

$\overset{\frown}{QPS} - \overset{\frown}{PS} = \overset{\frown}{RSP} - \overset{\frown}{PS}$

$\Rightarrow \overset{\frown}{PQ} = \overset{\frown}{SR}$

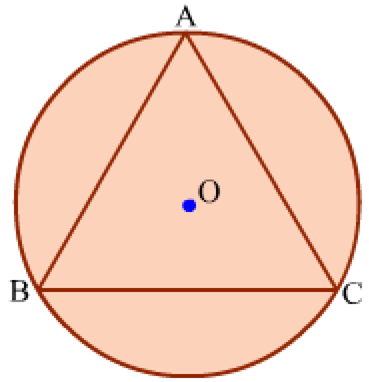
$\Rightarrow \overset{\frown}{PQ} \cong \overset{\frown}{SR}$

We know that if two arcs are congruent, then their corresponding chords are equal.

∴ PQ = SR

**Medium**

**Example 1:** An equilateral triangle ABC is inscribed in the given circle with centre O and radius 7 cm. Find the lengths of arcs AB, BC and CA.




**Solution:**

It is given that ΔABC is equilateral.

∴ AB = BC = CA

We know that if two chords of a circle are equal, then their corresponding arcs are congruent.

$\therefore \overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CA}$

Thus, points A, B and C divide the circle into three equal parts.

$\Rightarrow \overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CA} = \frac{1}{3} \times \text{Circumference}$

Now,

Circumference = $2\pi r$

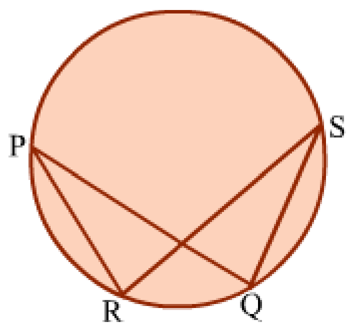
$\Rightarrow$ Circumference = $2 \times \frac{22}{7} \times 7\,\text{cm}$

$\Rightarrow$ Circumference = 44 cm

$\Rightarrow \overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CA} = \frac{44}{3}\,\text{cm}$

$\Rightarrow \overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CA} = 14\frac{2}{3}\,\text{cm}$

**Example 2:** If PQ and SR are two equal chords of the given circle, then show that PR and SQ are also equal chords.




**Solution:** It is given that PQ and SR are equal chords of the given circle. We know that if two chords of a circle are equal, then their corresponding arcs are congruent.

$\therefore \overset{\frown}{PRQ} \cong \overset{\frown}{SQR}$

$\Rightarrow \overset{\frown}{PRQ} = \overset{\frown}{SQR}$

On subtracting $\overset{\frown}{RQ}$ from both sides, we get:

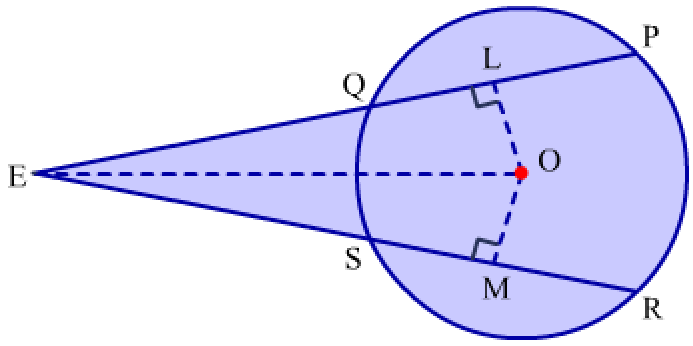
$\overset{\frown}{PR} = \overset{\frown}{SQ}$

By using the converse of the above property, we get:

PR = SQ

**Hard**

**Example 1**: In the given circle, arcs PQ and RS are congruent. Show that SE = QE.




**Solution**:

We know that equal chords are equidistant from the centre of a circle.

∴ OL = OM

In ΔOLE and ΔOME, we have:

∠OLE = ∠OME = 90° (∵ OL and OM are perpendiculars)

OL = OM (Proved above)

OE = OE (Common side)

∴ ΔOLE ≅ ΔOME (By the RHS congruence rule)

⇒ LE = ME… (1) [By CPCT]

We know that if two arcs are congruent, then their corresponding chords are equal. Since arcs PQ and RS are congruent, chords PQ and RS are equal.

$\Rightarrow \frac{1}{2} PQ = \frac{1}{2} RS$

⇒ QL = SM … (2) [∵ Perpendicular from centre to chord bisects chord]

On subtracting equation (2) from equation (1), we get:

LE − QL = ME − SM

⇒ QE = SE

**Angles Subtended by Congruent Arcs**

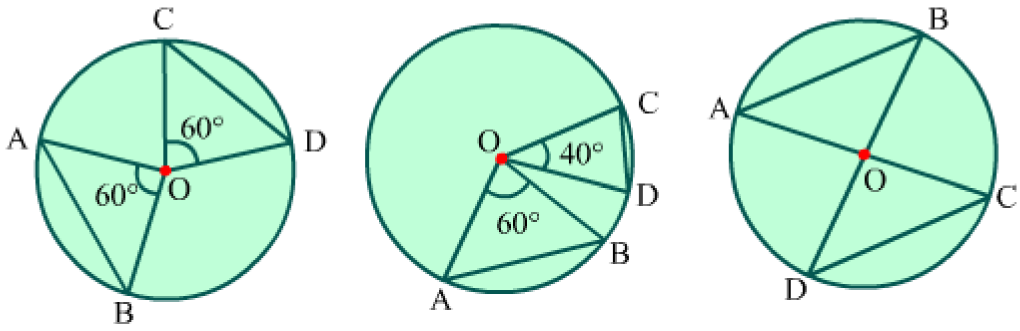
The given figure shows five children playing ball in a circular park. They are positioned at points A, B, C, D and E. The child standing at point A is at the centre of the circle, while the others are at its circumference.


The lines joining the different points represent the paths followed by the ball during play. Note how paths BC and DE act as chords of the circle. If we assume that the distances BC and DE are equal, then the arcs corresponding to them will also be equal or congruent. Clearly, the angles subtended at centre A by arcs BC and DE are equal, i.e., ∠BAC and ∠DAE are equal. By this, we can conclude that congruent arcs subtend equal angles at the centre of a circle. Let us understand this property and solve some problems based on it.

**Solved Examples**

**Easy**

**Example 1**: For each figure, state whether or not the arcs AB and CD are equal.




**(i)** **(ii)** **(iii)**

**Solution**:

We know that arcs subtending equal angles at the centre of a circle are congruent.

**In figure (i)**:

∠AOB = ∠COD = 60°

⇒ Arc AB = Arc CD

**In figure (ii)**:

∠AOB = 60° and ∠COD = 40°

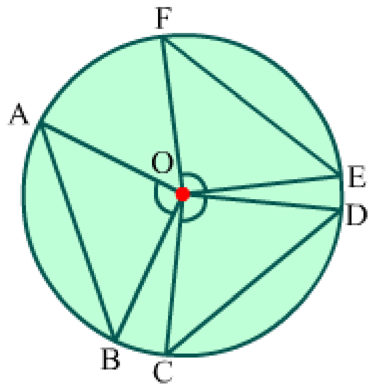
⇒ Arc AB ≠ Arc CD

**In figure (iii)**:

∠AOB = ∠COD (Vertically opposite angles)

⇒ Arc AB = Arc CD

**Example 2**: In the given circle with centre O, arcs AB and CD are equal and arcs AB and EF are equal. If ∠AOB = 55°, then find the measure of ∠FOE and ∠COD.




**Solution**:

It is given that:

Arc AB = Arc CD

Arc AB = Arc EF

∴ Arc CD = Arc EF

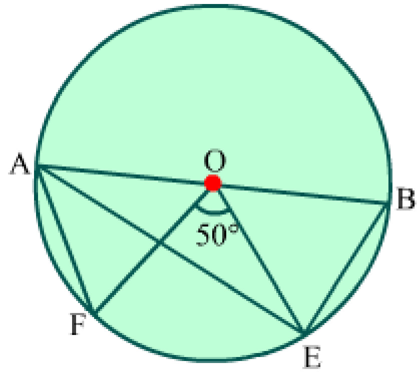
We know that congruent arcs subtend equal angles at the centre of a circle.

∴ ∠AOB = ∠COD = ∠EOF

Now, ∠AOB = 55°

∴ ∠COD = ∠EOF = 55°

**Example 3**: In the given circle with centre O, AF = BE and ∠FOE = 50°. Find the measure of ∠AOF.




**Solution**:

It is given that:

Chord AF = Chord BE

We know that congruent arcs subtend equal angles at the centre of a circle.

∴ ∠AOF = ∠BOE ... (1)

AOB is a straight line; therefore, we have:

∠AOF + ∠FOE + ∠BOE = 180°

⇒ 2∠AOF + 50° = 180° (By equation 1)

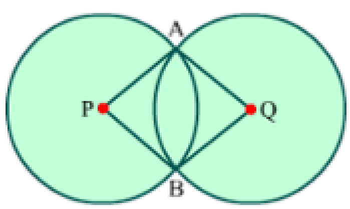
⇒ 2∠AOF = 180° − 50°

⇒ 2∠AOF = 130°

⇒ ∠AOF = 65°

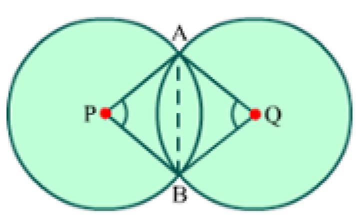
**Medium**

**Example 1**: The given figure shows two congruent circles with centres P and Q, and intersecting each other at points A and B. Show that ∠APB = ∠AQB.




**Solution**:

***Construction***: Draw a chord AB that is common to the given circles.




We know that if two chords of congruent circles are equal, then their corresponding arcs are congruent.

We have AB as the common chord of the two given circles. Therefore, the length of arc AB is the same in both circles.

We know that congruent arcs subtend equal angles at the centre of a circle.

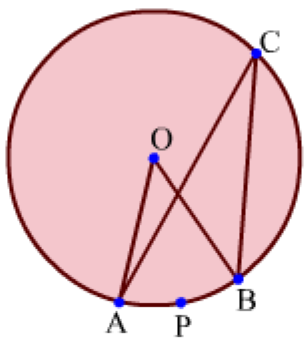
∴ ∠APB = ∠AQB

**Relation between Angles Subtended by an Arc at The Centre and Anywhere on the Circle**

**Observing the Angles Subtended by an Arc at the Centre and on the Circle**

We know that an infinite number of points lie on the circumference of a circle. The portion of circumference between any two such points is known as an **arc**. Every arc subtends an angle at the centre and a particular angle at any point on the circle.

Let us consider any angle ∠ACB inscribed in the major arc ACB of a circle having centre at point O as shown below.




It can be seen that the arc APB is intercepted by∠ACB.

Also, the arc APB subtends ∠AOB at the centre. Thus, ∠AOB is the measure of arc APB.

In other words, ∠AOB and ∠ACB are subtended by the same arc APB at the centre O and at any point C on the circle respectively.

There is a relation between ∠AOB (measure of intercepted arc) and ∠ACB (inscribed angle).

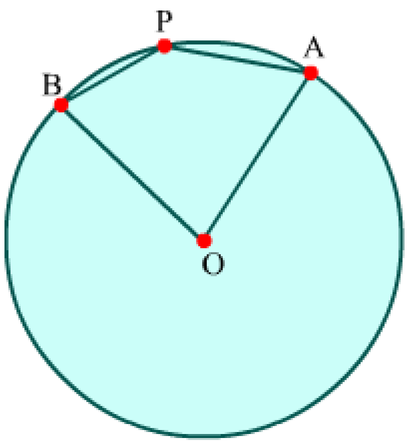
In this lesson, we will learn the theorem defining the relation between these two types of angles. We will also solve some examples related to the same.

**Know More**

The relation between the angles subtended by an arc at the centre and on the circumference of a circle is known as the **central angle theorem**.

This relation holds true only when the inscribed angle (i.e., the angle subtended at the circumference) is in the major arc. If, however, the inscribed angle is in the minor arc (as is ∠BPA in the following figure), then its relation with the central angle (i.e., the angle at the centre) is given by the formula:

$$\text{Inscribed angle} = 180° - \left(\frac{\text{Central angle}}{2}\right)$$

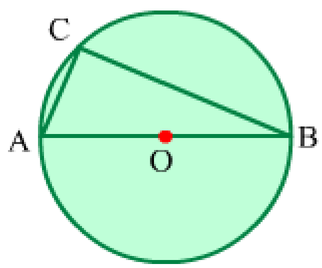



**Angle in a Semicircle is a Right Angle**

**Statement**: Angle in a semicircle is a right angle.

**Given**: A circle with centre O and diameter AB

**To prove**: ∠ACB = 90°




**Proof**: We know that the angle subtended by an arc at the centre of a circle is twice the angle subtended by it at the circumference of the circle.

∴ ∠AOB = 2∠ACB

$\Rightarrow 2\angle ACB = 180°$ ($\because$ AOB is a straight line)

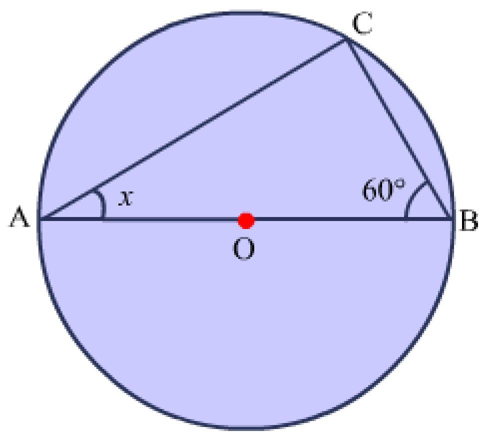
$\Rightarrow \angle ACB = 90°$

Now, AB is the diameter of the circle and it divides the circle into two semicircles. ∠ACB is inscribed in the semicircle. Hence, an angle in a semicircle is a right angle.

**Solved Examples**

**Easy**

**Example 1:** Find the value of $x$ in the given circle with centre O and diameter AB.




**Solution:**

We know that an angle in a semicircle is a right angle.

$\therefore \angle ACB = 90°$ ($\because$ AB is the diameter of the circle)

On using the angle sum property in ΔACB, we get:

$\angle ACB + \angle CBA + \angle BAC = 180°$

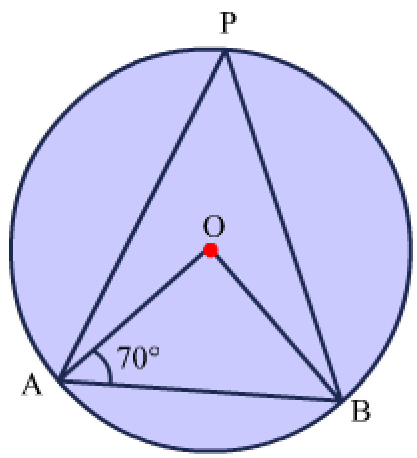
$\Rightarrow 90° + 60° + x = 180°$

$\Rightarrow 150° + x = 180°$

$\Rightarrow x = 180° - 150°$

$\Rightarrow x = 30°$

**Example 2:** Find the measure of ∠APB in the given circle.




**Solution:**

In ΔOAB, we have:

OA = OB (Radii of the circle)

⇒ ∠OBA = ∠OAB = 70° (∵ Angles opposite equal sides are equal)

Using the angle sum property in ΔOAB, we get:

∠OAB + ∠OBA + ∠AOB = 180°

⇒ 70° + 70° + ∠AOB = 180°

⇒ 140° + ∠AOB = 180°

⇒ ∠AOB = 180° − 140°

⇒ ∠AOB = 40°

We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle. In the given circle, arc AB subtends ∠AOB at the centre and ∠APB at the circumference.

So, ∠AOB = 2∠APB

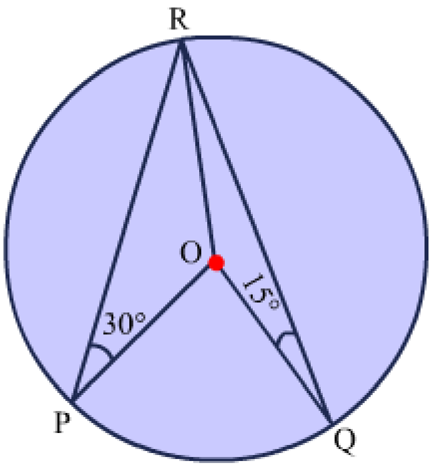
⇒ 40° = 2∠APB

⇒ ∠APB = $\frac{40°}{2}$

⇒ ∠APB = 20°

**Medium**

**Example 1:** Find the measure of ∠POQ in the given circle.




**Solution:**

In the given circle, □OPR = 30° and □OQR = 15°.

In ΔOPR, we have:

OP = OR (Radii of the circle)

□ □ORP = □OPR = 30° (∵ **Angles opposite equal sides are equal**)

Similarly, we can find that □ORQ = □OQR = 15°

Now, □PRQ = □ORP + □ORQ

□ □PRQ = 30° + 15°

□ □PRQ = 45°

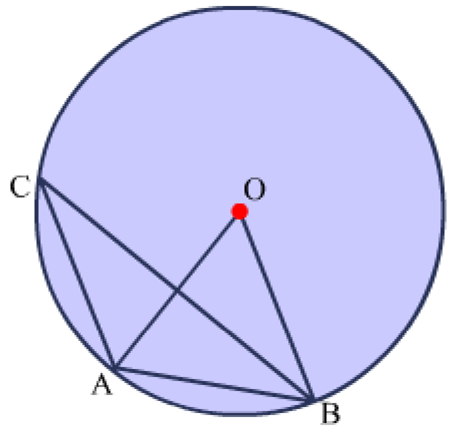
We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle. In the given circle, arc PQ subtends ∠POQ at the centre and ∠PRQ at the circumference.

So, ∠POQ = 2∠PRQ

⇒ ∠POQ = 2 × 45°

⇒ ∠POQ = 90°

**Example 2:** In the given circle with centre O, chord AB is equal to the radius of the circle. Find the measure of ∠ACB.




**Solution:**

It is given that chord AB is equal to the radius of the circle.

So, AB = OA = OB   (∵ OA and OB are radii of the circle)

Thus, ΔOAB is equilateral.

∴ ∠AOB = 60°  (∵ Each angle of an equilateral triangle measures 60°)

We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle. In the given circle, arc AB subtends ∠AOB at the centre and ∠ACB at the circumference.

So, ∠AOB = 2∠ACB

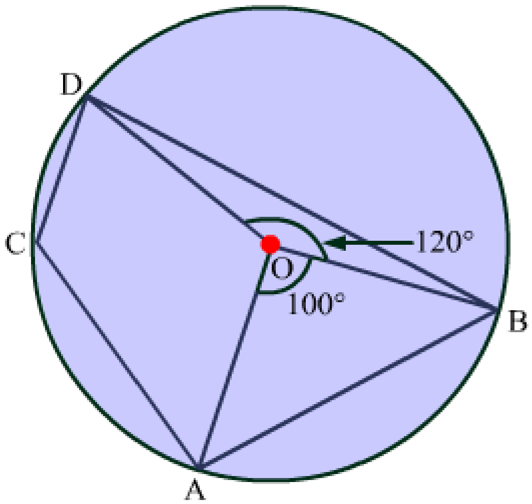
⇒ 60° = 2∠ACB

⇒ ∠ACB = $\frac{60°}{2}$

⇒ ∠ACB = 30°

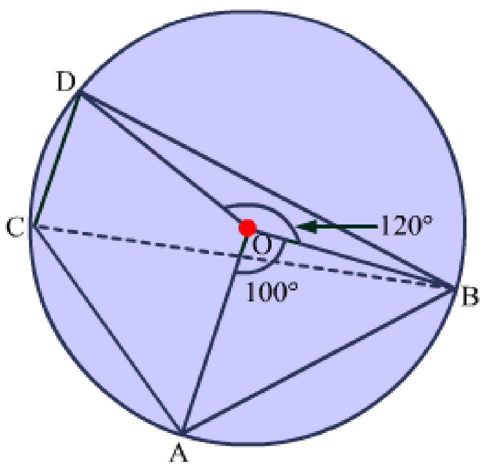
**Hard**

**Example 1**: Find the measure of ∠ACD in the given circle.




**Solution**:

***Construction***: Join B to C.




We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle.

In the given circle, arc AB subtends ∠AOB at the centre and ∠ACB at the circumference.

So, ∠AOB = 2∠ACB

⇒ 100° = 2∠ACB

⇒ $\angle ACB = \frac{100^\circ}{2}$

⇒ ∠ACB = 50°

Also, arc BD subtends ∠BOD at the centre and ∠BCD at the circumference.

So, ∠BOD = 2∠BCD

⇒ 120° = 2∠BCD

⇒ ∠BCD = $\frac{120°}{2}$

⇒ ∠BCD = 60°

Now, ∠ACD = ∠ACB + ∠BCD

⇒ ∠ACD = 50° + 60°

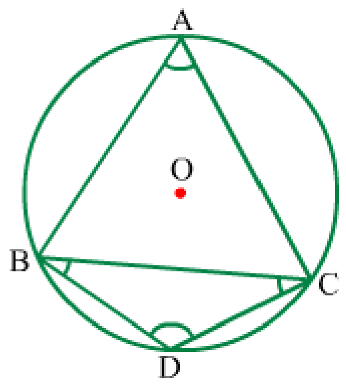
⇒ ∠ACD = 110°

Angles in The Same Segment of a Circle

**Angles in the Major and Minor Segments**

We know that the chord of a circle divides it into two regions. These regions are called **segments of the circle** and are classified as the **major segment** and the **minor segment**.

Observe the given circle.




In this circle, ∠BAC lies in the major segment whereas ∠BDC lies in the minor segment. It can be seen that ∠BAC is an acute angle while ∠BDC is an obtuse angle.

So, it can be concluded that the angle lying in the major segment is an acute angle and the angle lying in the minor segment is an obtuse angle. This statement is true for all major and minor segments in a circle.

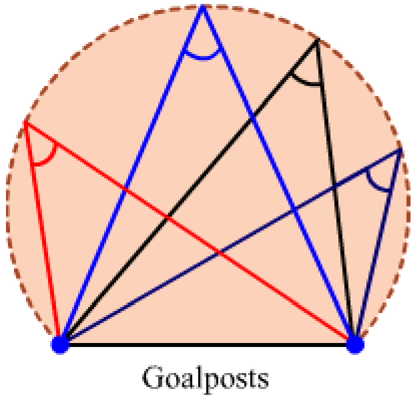
**There is no relation between angles in different segments, but what about the angles in the same segment?**

In this lesson, we will learn about the angles in the same segment of a circle and the relation between them. We will also solve some examples dealing with the same.

**Did You Know?**

**Angle for scoring a goal in soccer**

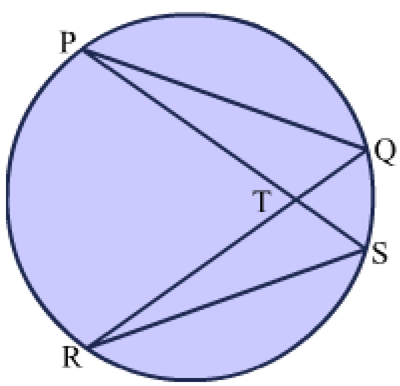
The angle of every possible shot to score a goal is constant for all positions on the same arc of a circle; however, the distance of a shot changes with change in position.




**Solved Examples**

**Easy**

**Example 1**: In the given circle, chords PQ and RS are equal and chords PS and QR intersect at point T. Show that PT = RT and TQ = TS.




**Solution**:

In ΔPQT and ΔRST, we have:

PQ = RS(Given)

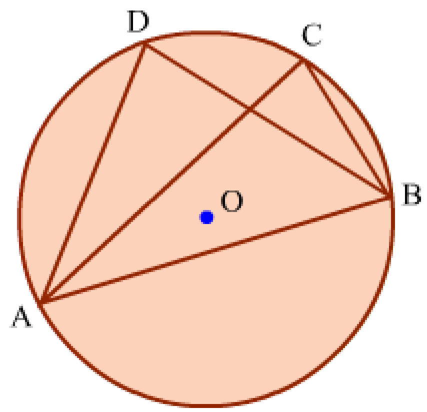
∠TPQ = ∠TRS(∵ Angles in the same segment of a circle are equal)

Similarly, ∠TQP = ∠TSR

∴ ΔPQT ≅ ΔRST(By the ASA congruence criterion)

⇒ PT = RT and TQ = TS (By CPCT)

**Example 2**: In the given circle, find the value of ∠DAB if ∠BCA = 80° and DA = DB.




**Solution**:

From the figure, we have:

∠BCA = ∠BDA = 80° (∵ Angles in the same segment of a circle are equal)

DA = DB (Given)

⇒ ∠DBA = ∠DAB ... (1) [∵ Angles opposite equal sides are equal]

On using the angle sum property in ΔADB, we get:

∠DAB + ∠DBA + ∠BDA = 180°

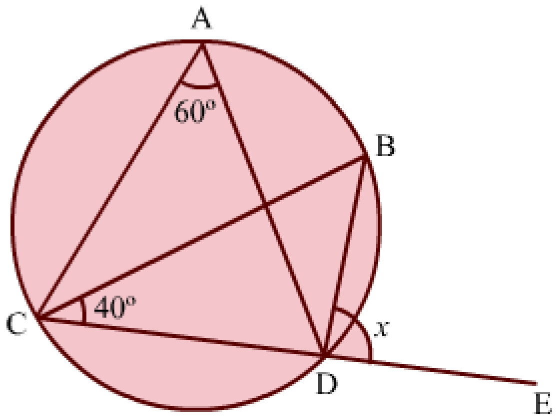
⇒ 2∠DAB + 80° = 180° (By equation 1)

⇒ 2∠DAB = 180° – 80°

⇒ 2∠DAB = 100°

⇒ ∠DAB = 50°

**Example 3**: What is the value of *x* in the given figure?




**Solution**:

We know that angles in the same segment are equal.

∴ ∠CAD = ∠CBD = 60°

Now, ∠BDE is an exterior angle of ΔBCD.

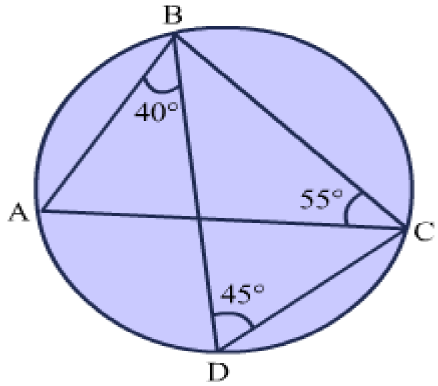
So, ∠BDE = ∠CBD + ∠DCB (∵ Exterior angle equals sum of interior opposite angles)

⇒ $x = 60° + 40°$

⇒ $x = 100°$

**Medium**

**Example 1**: What are the measures of ∠BAC, ∠ACD, ∠ABC, and ∠DBC?




**Solution**:

We know that angles in the same segment are equal.

So, ∠BAC = ∠BDC = 45°

Similarly, ∠ABD = ∠ACD = 40°

On using the angle sum property in ΔABC, we get:

∠ABC + ∠BAC + ∠ACB = 180°

⇒ ∠ABC + 45° + 55° = 180°

⇒ ∠ABC + 100° = 180°

⇒ ∠ABC = 180° – 100°

⇒ ∠ABC = 80°

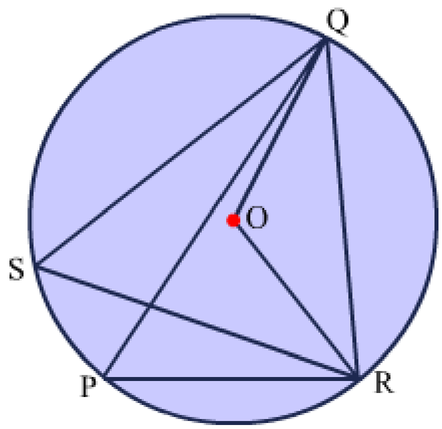
Now, ∠ABC = ∠ABD + ∠DBC

⇒ 80° = 40° + ∠DBC

⇒ ∠DBC = 80° – 40°

⇒ ∠DBC = 40°

**Example 2**: In the given circle with centre O, ∠PQR = 37° and ∠QRP = 83°. What are the measures of ∠RSQ and ∠ROQ?




**Solution**:

On using the angle sum property in ΔPQR, we get:

$\angle PQR + \angle QRP + \angle RPQ = 180°$

$\Rightarrow 37° + 83° + \angle RPQ = 180°$

$\Rightarrow 120° + \angle RPQ = 180°$

$\Rightarrow \angle RPQ = 180° - 120°$

$\Rightarrow \angle RPQ = 60°$

We know that angles in the same segment are equal.

$\therefore \angle RPQ = \angle RSQ = 60°$

We also know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle.

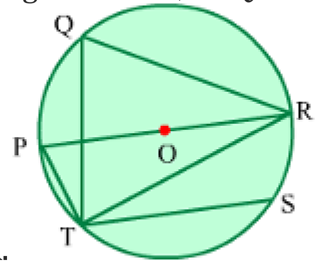
So, $\angle ROQ = 2\angle RSQ$

$\Rightarrow \angle ROQ = 2 \times 60°$

$\Rightarrow \angle ROQ = 120°$

**Hard**

**Example 1**: In the given circle, ∠TQR = 70° and PR is the diameter. If TS||PR, then find the measure of ∠STR.




**Solution**:

We know that angles in the same segment are equal.

∴ ∠TQR = ∠TPR = 70°

We also know that an angle in a semicircle is a right angle.

∴ ∠RTP = 90° (∵ PR is the diameter)

On using the angle sum property in ΔRTP, we obtain:

∠TPR + ∠RTP + ∠PRT = 180°

⇒ 70° + 90° + ∠PRT = 180°

⇒ 160° + ∠PRT = 180°

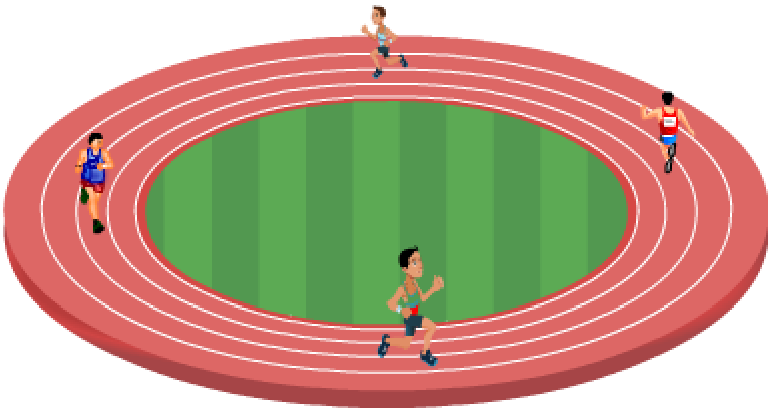
⇒ ∠PRT = 20°

Now, PR||TS and RT is the transversal.

So, ∠STR = ∠PRT = 20° (∵ Alternate angles are equal)

**Concyclic Points**

You must have seen circular running tracks like the one shown below. You can see concentric circles that divide the track into different paths. Four runners can be seen at different positions on the track.



Note how the runners are on the circumference of the same circle on the track. Things which lie on the same circle are referred to as concyclic; so, these runners are also concyclic. Let us go through this lesson to know more about concyclic points and the theorem related to them.

**Know More**

A set of more than four points is concyclic if and only if every four-point subset is concyclic. This property is the analogue of concyclicity.

**Whiz Kid**

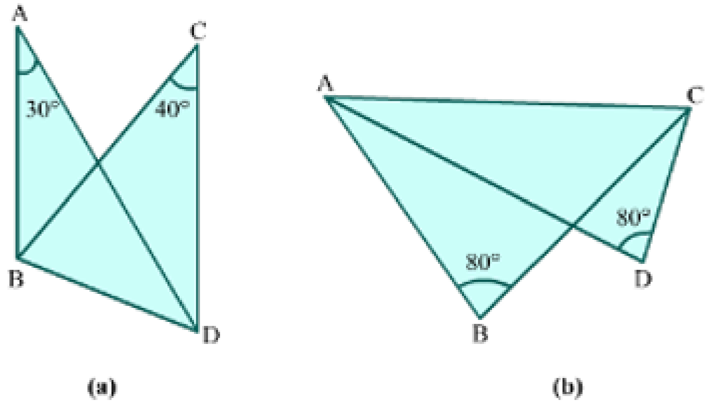
**General condition for concyclicity**

If $n$ distinct points lie on a circle and we join any two points, then $\frac{n(n-1)}{2}$ perpendicular bisectors of the line segments should meet at a single point that is the centre of the circle.

**Solved Examples**

**Easy**

**Example 1**: **For each figure, state whether or not the points A, B, C and D are concyclic.**




**Solution**:

**In figure (a)**:

Points A and C are present on the same side of the line joining points B and D.

We have ∠BAD = 30° and ∠BCD = 40°.

Clearly, line segment BD does not subtend equal angles at points A and C that lie on the same side of BD. Therefore, points A, B, C and D do not lie on a circle, i.e., they are not concyclic.

**In figure (b)**:

Points B and D are present on the same side of the line joining points C and A.

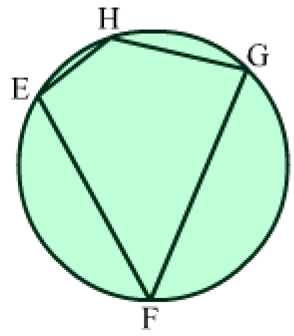
We have ∠ABC = ∠ADC = 80°.

Clearly, line segment CA subtends equal angles at points B and D that lie on the same side of CA. Therefore, points A, B, C and D lie on a circle, i.e., they are concyclic.

**Properties of Cyclic Quadrilaterals**

**Cyclic Quadrilaterals**

We know that points lying on the same circle are called concyclic points. Let us consider four concyclic points, say E, F, G and H, and the circle passing through them. If we join the four points, then we get a quadrilateral as is shown in the figure below.




A quadrilateral whose vertices lie on a circle or through whose vertices it is possible to draw a circle is known as a **cyclic quadrilateral**. In the given figure, the vertices E, F, G and H lie on a circle; hence, EFGH is a cyclic quadrilateral. The circle on which the quadrilateral lies is called a circumcircle.

Cyclic quadrilaterals are a little different from regular quadrilaterals as they exhibit a few special properties. In this lesson, we will discuss these properties of cyclic quadrilaterals and solve some problems based on them.

**Did You Know?**

If a cyclic quadrilateral has unequal rational sides in either arithmetic or geometric progression, then there does not exist any cyclic quadrilateral with rational area.

**Know More**

- A cyclic quadrilateral is also called **chordal quadrilateral** because the sides of the quadrilateral are chords of the circumcircle. Another name for this quadrilateral is **concyclic quadrilateral**.

- If the opposite sides of a cyclic quadrilateral are extended to meet, say at points E and F, then the internal angle bisectors of the angles formed at points E and F are perpendicular.

- The opposite sides and the diagonals of a cyclic quadrilateral ABCD are related as: AC.BD = AD.BC + AB.CD. This relationship is known as **Ptolemy's theorem**.

- The area of a cyclic quadrilateral is $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where *a*, *b*, *c* and *d* are the lengths of the sides of the cyclic quadrilateral and $s=\dfrac{a+b+c+d}{2}$.

**Whiz Kid**

- In a cyclic quadrilateral ABCD with circumcentreO, if the diagonals AC and BD intersect at point P, then ∠APB is the arithmetic mean of ∠AOB and ∠COD.
- Four line segments are concurrent if each is perpendicular to one side of a cyclic quadrilateral and passes through the midpoint of the opposite side. These line segments are called **maltitudes**, which means 'midpoint altitudes'.

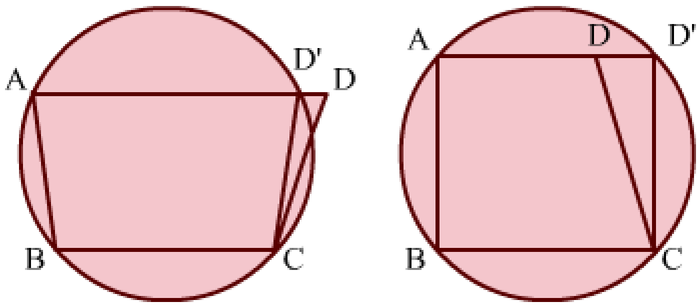
**Proving the Converse of Property**

**Statement**: If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

**Given:** A quadrilateral ABCD with ∠ABC + ∠ADC = 180° and ∠BAD + ∠BCD = 180°

**To prove:** ABCD is a cyclic quadrilateral.

**Proof:** Let us assume that ABCD is not a cyclic quadrilateral. Suppose a circle passes through the three non-collinear points A, B and C and meets AD or AD produced, at D′.




Now, on joining D′ to C, we get the cyclic quadrilateral ABCD′.

In ABCD′, we have:

∴∠ABC + ∠AD′C = 180° (∵ Opposite angles of a cyclic quadrilateral are supplementary)

But ∠ABC + ∠ADC = 180° (Given)

∴ ∠AD′C = ∠ADC, which can be possible only if D and D′ coincide

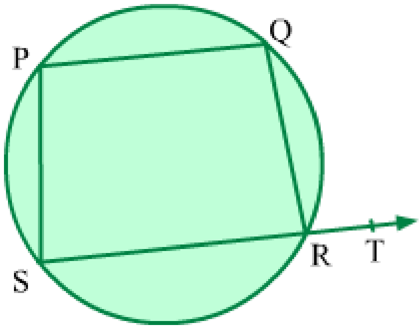
Thus, the circle passing through points A, B and C also passes through point D. Therefore, ABCD is a cyclic quadrilateral.

**Proving that the Exterior Angle of a Cyclic Quadrilateral Is Equal to the Interior Opposite Angle**

**Statement:** The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

**Given:** A cyclic quadrilateral PQRS with side SR extended up to point T

**To prove:** ∠QRT = ∠QPS.




**Proof**: We know that the opposite angles of a cyclic quadrilateral are supplementary.

∴ ∠QPS + ∠QRS = 180° … (1)

Also, ∠QRT + ∠QRS = 180°… (2) [Linear pair of angles]

From equations (1) and (2), we obtain:

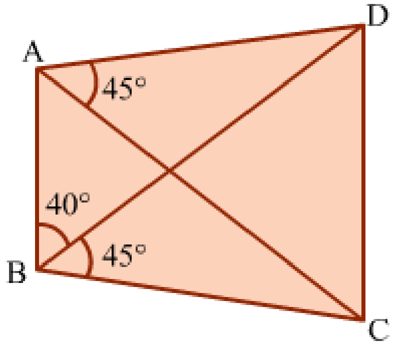
∠QPS + ∠QRS = ∠QRT + ∠QRS

⇒ ∠QPS = ∠QRT

**Solved Examples**

**Easy**

**Example 1**: What is the measure of ∠ADC in the given figure?



**Solution**:

In the figure, ∠CAD = ∠CBD = 45°

We know that if a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, then the four points are concyclic.

Therefore, A, B, C and D are concyclic points and ABCD is a cyclic quadrilateral.

We know that in a cyclic quadrilateral, opposite angles are supplementary.

So, ∠ABC + ∠ADC = 180°

⇒ (40° + 45°) + ∠ADC = 180°

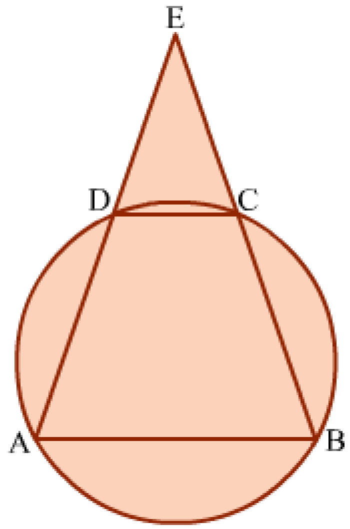
⇒ 85° + ∠ADC = 180°

⇒ ∠ADC = 180° − 85°

⇒ ∠ADC = 95°

**Example 2**: In ΔAEB, AE = BE. A circle passing through points A and B intersects AE and BE at points D and C respectively. Prove that the line segment DC is parallel to AB.

**Solution**: The figure for the given problem can be drawn as is shown.




In ΔAEB, we have:

AE = BE  (Given)

⇒ ∠EBA = ∠EAB... (1)  [∵ Angles opposite equal sides of a triangle are equal]

Now, ABCD lies on a circle; so, it is a cyclic quadrilateral. We know that the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

So, ∠EDC = ∠CBA

⇒ ∠EDC = ∠EBA ... (2)  [∵∠CBA = ∠EBA]

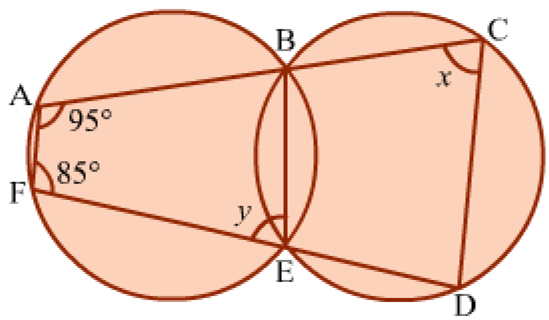
From equations (1) and (2), we obtain:

∠EDC = ∠EAB

We can see that line segments DC and AB are cut by the transversal EA. ∠EDC and ∠EAB are equal corresponding angles. Therefore, by the converse of the corresponding angles axiom, we can say that DC is parallel to AB.

**Medium**

**Example 1**: Find the values of *x* and *y* in the given figure.




**Solution**:

In the figure, we have two cyclic quadrilaterals ABEF and BCDE.

In ABEF, we have:

∠BAF + ∠BEF = 180° (∵ Opposite angles of a cyclic quadrilateral are supplementary)

⇒ 95° + $y$ = 180°

⇒ $y$ = 180° − 95°

⇒ $y$ = 85°

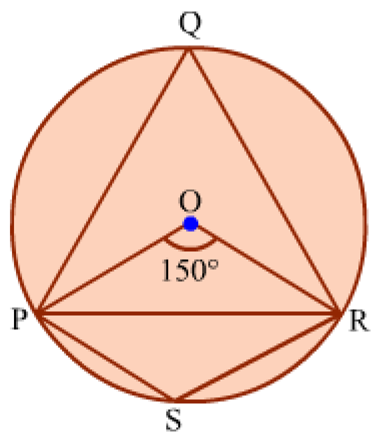
In BCDE, we have:

∠BEF = ∠BCD (∵ Exterior angle of a cyclic quadrilateral equals interior opposite angle)

⇒ $y = x$

⇒ $x = 85°$

**Example 2**: What is the measure of ∠PSR in the given figure?




**Solution**: We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle.

So, ∠POR = 2∠PQR

⇒ $\angle PQR = \frac{1}{2} \angle POR$

⇒ $\angle PQR = \frac{1}{2} \times 150°$

⇒ ∠PQR = 75° ... (1)

Now, quadrilateral PQRS is cyclic.

So, ∠PQR + ∠PSR = 180° (∵ Opposite angles of a cyclic quadrilateral are supplementary)

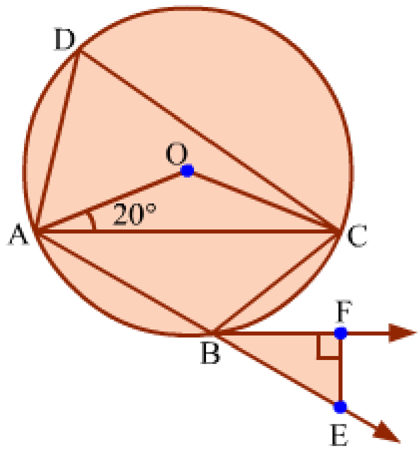
⇒ 75° + ∠PSR = 180° (By equation 1)

⇒ ∠PSR = 180° − 75°

⇒ ∠PSR = 105°

**Hard**

**Example 1**: In the given figure, find the value of ∠BEF if BF is the bisector of ∠CBE.




**Solution**:

In ΔOAC, we have:

OA = OC (Radii of the circle)

⇒ ∠OCA = ∠OAC = 20° (∵ Angles opposite equal sides of a triangle are equal)

On using the angle sum property in ΔOAC, we obtain:

∠AOC + ∠OAC + ∠OCA = 180°

⇒ ∠AOC + 20° + 20° = 180°

⇒ ∠AOC + 40° = 180°

⇒ ∠AOC = 180° – 40°

⇒ ∠AOC = 140°

We know that the angle subtended by an arc at the centre of a circle is double the angle subtended by it at the circumference of the circle.

So, ∠AOC = 2∠ADC

$\Rightarrow \angle ADC = \frac{1}{2} \angle AOC$

$\Rightarrow \angle ADC = \frac{1}{2} \times 140°$

$\Rightarrow \angle ADC = 70°$

Since ABCD is a cyclic quadrilateral, we have:

$\angle CBE = \angle ADC$ ($\because$ Exterior angle of a cyclic quadrilateral equals interior opposite angle)

$\Rightarrow \angle CBE = 70°$

It is given that BF bisects $\angle CBE$.

So, $\angle EBF = \frac{1}{2} \angle CBE$

$\Rightarrow \angle EBF = \frac{1}{2} \times 70°$

$\Rightarrow \angle EBF = 35°$

On using the angle sum property in $\Delta BEF$, we obtain:

$\angle EBF + \angle BEF + \angle BFE = 180°$

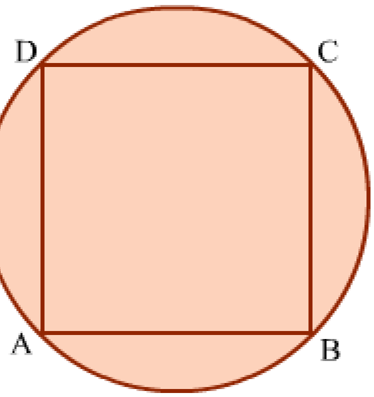
$\Rightarrow 35° + \angle BEF + 90° = 180°$

$\Rightarrow \angle BEF + 125° = 180°$

$\Rightarrow \angle BEF = 180° - 125°$

$\Rightarrow \angle BEF = 55°$

**Example 2**: If points A, B, C and D divide the circumference of the given circle into four equal parts, then show that ABCD is a square.




**Solution**:

It is given that points A, B, C and D divide the circle into four equal parts.

∴ Arc AB = Arc BC = Arc CD = Arc DA

We know that if the arcs in a circle are congruent, then their corresponding chords are equal.

∴ Chord AB = Chord BC = Chord CD = Chord DA

Thus, all sides of quadrilateral ABCD are equal in length. Therefore, ABCD is a rhombus.

Points A, B, C and D are concyclic; so, ABCD is a cyclic quadrilateral.

Now, we know that in a cyclic quadrilateral, opposite angles are supplementary.

So, $\angle BAD + \angle BCD = 180°$

But $\angle BAD = \angle BCD$ (∵ Opposite angles of a rhombus are equal)

$\Rightarrow 2\angle BAD = 180°$

$\Rightarrow \angle BAD = 90°$

Similarly, $\angle ABC$, $\angle BCD$ and $\angle CDA$ measure 90°.

Hence, ABCD is a square.